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The Convergence of Cluster Expansion for Continuous Systems with Many-Body Interaction

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The existence of a unique thermodynamic state for dilute classical systems is proved for a class of multi-particle potentials under ordinary assumptions of stability and integrability. Thus we do not use the cumbersome conditions of regularity needed in previous publications for the many-body analysis. The method relies on the Poisson measure representation and cluster expansion for distribution functions.

KEY WORDS: Thermodynamic limit; cluster expansion; Poisson measure; many-body interaction.

INTRODUCTION

The Kirkwood-Salsburg equation method, well developed for a two-body interaction (see refs. 1 and 2), has also been applied for the investigation of classical continuous systems with a multi-particle potential in refs. 3 and 4. But, in addition to the usual stability condition (2.3), these applications for many-body potentials demand cumbersome regularity conditions. On the other hand, Brydges and Federbush,[(5)] see also ref. 6, proposed a very elegant method of constructing the Mayer series which at the same time leads to essential simplifications in the proof of its convergence. But, unfortunately, an application of this method to the many-body interaction gives rise to certain difficulties because there are too many tree graphs in this context, see remark in ref. 6. Later, in ref. 7, the Borel summability of the Brydges-Federbush-Mayer expansion for many-body potentials was proved.

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In this article we prove convergence of the Brydges-Federbush type cluster expansion,[8] see also ref. 9, for dilute continuous systems of classical statistical mechanics with the many-body interaction satisfying ordinary assumptions of stability and integrability. To prove this we use the Poisson measure representation of correlation functions which was proposed in ref. 10 and developed in refs. 11–13. This integral representation, which from the physical point of view is exactly the integral over the densities of ideal gas, allows us to apply a technique which is very close to the treatment of lattice systems.

A short contents of this article is the following. In Section 1 we briefly state some notions and formulae of the Poisson analysis needed for the later exposition. In Section 2 we obtain a representation for correlation functions of classical systems with a many-body potential by the Poisson integrals. In Section 3 we construct cluster expansions and formulate the basic results of the paper. And finally, in Section 4 we prove the convergence of the cluster expansions and other results of the paper.

1. SOME REMARKS ON THE POISSON ANALYSIS

A detailed exposition of different aspects of the Poisson analysis may be found in refs. 14–21, see also introductory sections of refs. 11, 13, 22. In this section we are going to remind only some of the most useful definitions and formulae which will be used later.

Definition 1. For any measurable $\Lambda \subseteq \mathbb{R}^3$, the Poisson measure $P_z^\Lambda(\cdot)$ on the Borel σ-algebra on the Schwartz space of tempered distributions $S'(\Lambda)$ endowed with the strong dual topology is defined by the following characteristic functional:

$$\int_{S'(\Lambda)} dP_z^\Lambda(q)\, e^{i\langle \phi, q\rangle} = \exp\left[z \int_\Lambda (e^{i\phi(x)} - 1)\, dx \right] \tag{1.1}$$

where $\varphi \in S(\Lambda)$, $\langle \cdot, \cdot \rangle$ denotes the pairing in the sense of $L_2(\Lambda, dx)$, and $z > 0$ is an intensity parameter.

Proposition 1. For any $F \in \mathbb{L}_2^P \equiv L_2(S'(\Lambda), dP_z^\Lambda)$ and $|\Lambda| < \infty$ the following formula is true:

$$\int_{S'(\Lambda)} dP_z^\Lambda(q)\, F[q] = e^{-z|\Lambda|} \sum_{n=0}^{\infty} \frac{z^n}{n!} \int_{\Lambda^n} (dx)_n^1\, F\left[\sum_{j=1}^{n} \delta_{x_j} \right] \tag{1.2}$$

where $(dx)_n^1 \equiv dx_1 \cdots dx_n$ and $\delta_x(\cdot) \equiv \delta(\cdot - x)$.

Remark. Because of Proposition 1, the reader who do not want to penetrate into the mathematical depths can always regard the integration over the Poisson measure P_z^Λ of an arbitrary functional F just as a convenient notation for the right-hand side of formula (1.2).

Using formula (1.2) it is easy to verify the following "cluster property" of the Poisson measure:

Proposition 2. For all measurable $X, X' \subset \Lambda$ such that $X \cap X' = \varnothing$ and for all $F, F' \in \mathbb{L}_2^P$

$$\int_{S'(\Lambda)} dP_z^\Lambda(q)\, F[q\chi_X]\, F'[q\chi_{X'}] = \int_{S'(\Lambda)} dP_z^\Lambda(q)\, F[q\chi_X] \int_{S'(\Lambda)} dP_z^\Lambda(q)\, F'[q\chi_{X'}] \tag{1.3}$$

and

$$\int_{S'(\Lambda)} dP_z^\Lambda(q)\, F[q\chi_X] = \int_{S'(X)} dP_z^X(q)\, F[q] \tag{1.4}$$

where χ_X is an indicator of the set X.

Next, defining the Wick regularization of a product of m Poisson fields by the following explicit formula:

$$:\prod_{j=1}^{n} q(x_j): = \prod_{j=1}^{n} \left(q(x_j) - \sum_{i=1}^{j-1} \delta(x_i - x_j) \right) \tag{1.5}$$

we claim that

$$\int_{S'(\Lambda)} dP_z^\Lambda(q) :\prod_{i=1}^{n} q(x_i): F[q] = z^n \int_{S'(\Lambda)} dP_z^\Lambda(q)\, F\left[q + \sum_{j=1}^{n} \delta_{x_j} \right] \tag{1.6}$$

holds in the weak sense for any $F \subset \mathbb{L}_2^P$ (see refs. 23, 24, 11, 13, 25).

Proposition 3. For averages of regularized products of the Poisson fields we have:

$$\int_{S'(\Lambda)} dP_z^\Lambda(q) :\prod_{i=1}^{m} q(x_i): = z^m \tag{1.7}$$

Finally, we will need the following generalized Wick theorem[13, 26] for the Poisson fields:

Theorem 1 (generalized Wick theorem). The product of regularized products of the Poisson fields is equal to the sum of all the corresponding regularized products with all possible "pairings" which connect the Poisson fields from the initially different regularized products including the regularized product without any "pairing", where the "pairing" of n Poisson fields is defined by the following formula:

$$\overbracket{q(x_1)\, q(x_2)\, \cdots q(x_n)} = q(x_1)\, \delta(x_1 - x_2) \cdots \delta(x_1 - x_n) \tag{1.8}$$

Examples.

$$\begin{aligned}
q(x_1)\, q(x_2)\, q(x_3) &= :q(x_1): \cdot :q(x_2): \cdot :q(x_3): \\
&= :q(x_1)\, q(x_2)\, q(x_3): + :\overbracket{q(x_1)\, q(x_2)}\, q(x_3): \\
&\quad + :q(x_1)\, \overbracket{q(x_2)\, q(x_3)}: + :\overbracket{q(x_1)\, q(x_2)\, q(x_3)}: \\
&\quad + :\overbracket{q(x_1)\, q(x_2)\, q(x_3)}:
\end{aligned}$$

$$\begin{aligned}
:q(x_1)\, q(x_2): \cdot :q(x_3): &= :q(x_1)\, q(x_2)\, q(x_3): + :\overbracket{q(x_1)\, q(x_2)\, q(x_3)}: \\
&\quad + :q(x_1)\, \overbracket{q(x_2)\, q(x_3)}:
\end{aligned}$$

Pay attention that the summands $:\overbracket{q(x_1)\, q(x_2)}\, q(x_3):$ and $:\overbracket{q(x_1)\, q(x_2)\, q(x_3)}:$ are absent on the right-hand side of the last formula because they contain "pairings" connecting fields from initially the first regularized product.

Corollary 1. As the number of all the possible "pairings" of n Poisson fields is less or equal than $n!$, it follows from the generalized Wick theorem, definition (1.8) and Proposition 3 that the average of a product of regularized products of the Poisson fields contains no more than $n!$ terms, where n is the total number of the Poisson fields in all the regularized products.

2. POISSON INTEGRAL REPRESENTATION FOR THE CORRELATION FUNCTIONS

Let us consider the system of classical identical particles contained in a certain finite volume $\Lambda \subset \mathbb{R}^3$ and interacting through the following many-body potential

$$\mathbf{V} \equiv (V_2(x_1, x_2), \ldots, V_M(x_1, \ldots, x_M)) \tag{2.1}$$

where $M \geqslant 2$ and fixed. For such a system the potential energy of n particles located at points $x_1, \dots, x_n$ is:

$$V(x)_n^1 = \sum_{p=2}^{M} \sum_{1 \leqslant i_1 < \cdots < i_p \leqslant n} V_p(x_{i_1}, \dots, x_{i_p}) \tag{2.2}$$

where by definition $(x)_n^1 = (x_1, \dots, x_n)$.

We assume that for any $p = 2, \dots, M$ the function $V_p(x_1, \dots, x_p)$ is measurable with respect to the usual Borel σ-algebra on $(\mathbb{R}^3)^p$ and the potential energy (2.2) satisfies the following stability condition:

$$\exists B \geqslant 0: \quad \forall n \in \mathbb{N}, \quad \forall (x)_n^1 \in (\mathbb{R}^3)^n, \qquad V(x)_n^1 \geqslant -Bn \tag{2.3}$$

Then the finite volume distribution functions of the grand canonical ensemble can be represented[1,2] as the following absolutely convergent series:

$$\rho_\Lambda(x)_m^1 = \Xi_\Lambda^{-1} \sum_{n=0}^{\infty} \frac{z^{m+n}}{n!} \int_{\Lambda^n} (dx)_{m+n}^{m+1} \exp[-\beta V(x)_{m+n}^1] \tag{2.4}$$

where β is the inverse temperature, z the activity and

$$\Xi_\Lambda = \sum_{n=0}^{\infty} \frac{z^n}{n!} \int_{\Lambda^n} (dx)_n^1 \exp[-\beta V(x)_n^1]$$

the grand partition function for the volume Λ.

In refs. 10 and 12 a representation of the function ρ_Λ by the Poisson integrals was obtained for a pair potential. Now we generalize that representation to the case of M-particle interaction.

Notation 1. Let for all $X, X^{(1)}, \dots, X^{(p)} \subset \mathbb{R}^3$, $\{X\}^k = \overbrace{X, \dots, X}^{k \text{ times}}$ and

$$V_p(q; X^{(1)}, \dots, X^{(p)}) = \int_{X^{(1)}} dx^{(1)} \cdots \int_{X^{(p)}} dx^{(p)} :q(x^{(1)}) \cdots q(x^{(p)}): V_p(x^{(1)}, \dots, x^{(p)})$$

Theorem 2. Let V be a measurable function satisfying stability condition (2.3), then the following formula is valid in the weak sense:

$$\rho_\Lambda(x)_m^1 = \Xi(\Lambda)^{-1} \int_{S'(\Lambda)} dP_z^\Lambda(q) :\prod_{i=1}^{m} q(x_i): \exp\{-\beta V_\Lambda[q]\} \tag{2.5}$$

where

$$V_\Lambda[q] = \sum_{p=2}^{M} \frac{1}{p!} V_p(q; \{\Lambda\}^p) \tag{2.6}$$

and

$$\Xi(\Lambda) = e^{-z|\Lambda|}\Xi_\Lambda = \int_{S'(\Lambda)} dP_z^\Lambda(q)\, e^{-\beta V_\Lambda[q]} \tag{2.7}$$

Proof. With the help of formula (1.5), the expression (2.2) for the potential energy of $(m+n)$ particles can be written in the following form:

$$V(x)_{m+n}^1 = V_\Lambda\left[\sum_{i=1}^{m} \delta_{x_i} + \sum_{i=m+1}^{m+n} \delta_{x_i}\right]$$

Making renormalization as in (2.7) and using formula (1.2), one can rewrite (2.4) as follows:

$$\rho_\Lambda(x)_m^1 = \Xi(\Lambda)^{-1} z^m \int_{S'(\Lambda)} dP_z^\Lambda(q) \exp\left\{-\beta V_\Lambda\left[\sum_{i=1}^{m} \delta_{x_i} + q\right]\right\} \tag{2.8}$$

Finally, the application of the generalized integration by parts formula (1.6) to (2.8) gives (2.5). ∎

Remark. As we consider (2.5) in the weak sense, it is better instead of (2.5) to write

$$\rho_\Lambda(\phi_m) = \Xi(\Lambda)^{-1} \int_{S'(\Lambda)} dP_z^\Lambda(q) \langle :q^{\otimes m}:, \phi_m\rangle \exp\{-\beta V_\Lambda[q]\} \tag{2.9}$$

where $\phi_m \in \mathscr{D}(\Lambda^m)$ with $\operatorname{supp} \phi_m \subseteq \{X_1\}^m \subset \{\Lambda\}^m$.

3. CLUSTER EXPANSION

Let us fill $\mathbb{R}^3$ with unit cubes Δ which are half-opened and half-closed in an arbitrary way such that they are disjoint, i.e., $\Delta \cap \Delta' = \varnothing$ if $\Delta \neq \Delta'$ and assuming that Λ and X_1 are unions of a finite number of such cubes construct finite sequences $\mathfrak{Y}_n = (Y_1,..., Y_n)$, $\mathfrak{X}_n = (X_1,..., X_n)$ of subsets of Λ in such a way that $Y_1 = X_1$ and for any $i \geqslant 2$, $X_i = X_{i-1} \cup Y_i$ with Y_i representing from one to $(M-1)$ different cubes from $\mathfrak{D}_{\Lambda \setminus X_{i-1}}$:

$$Y_i = \begin{cases} \Delta_i^1, & \Delta_i^1 \in \mathfrak{D}_{\Lambda \setminus X_{i-1}} \\ \Delta_i^1 \cup \Delta_i^2, & \Delta_i^1, \Delta_i^2 \in \mathfrak{D}_{\Lambda \setminus X_{i-1}} \text{ and } \Delta_i^1 \neq \Delta_i^2 \\ \cdots \\ \bigcup_{p=1}^{M-1} \Delta_i^p, & \Delta_i^1, \ldots, \Delta_i^{M-1} \in \mathfrak{D}_{\Lambda \setminus X_{i-1}} \text{ and } \Delta_k^1 \neq \Delta_l^2 \text{ for } k \neq l \end{cases}$$

Remark. Further on we always will consider only such subsets X of $\mathbb{R}^3$ which are unions of a finite number of cubes Δ, denoting the corresponding set of cubes with $\mathfrak{D}_X$.

Having in mind the structure of sets $Y_2, \ldots, Y_n$, under the sum over Y_i, $i = 2, \ldots, n$, we will understand the following expression:

$$\sum_{Y_i \subset \Lambda \setminus X_{i-1}} = \sum_{|Y_i| = 1}^{M-1} \sum_{\substack{\Delta_i^1, \ldots, \Delta_i^{|Y_i|} \subset \Lambda \setminus X_{i-1} \\ \Delta_i^k \neq \Delta_i^l, k \neq l}} \frac{1}{|Y_i|!} \tag{3.1}$$

where the factor $1/(|Y_i|!)$ appears from the fact that any permutation of the cubes $\Delta_i^1, \ldots, \Delta_i^{|Y_i|}$ do not alter the set Y_i.

Notation 2. For short let us also denote

$$\sum_{2,\ldots,n}^{\Lambda} = \sum_{Y_2 \subset \mathfrak{D}_{\Lambda \setminus X_1}} \cdots \sum_{Y_n \subset \mathfrak{D}_{\Lambda \setminus X_{n-1}}}$$

assuming as usual[2] that

$$\sum_{2,\ldots,1}^{\Lambda} = \prod_{i=2}^{1} \left(\sum_{Y_i \subset \mathfrak{D}_{\Lambda \setminus X_{i-1}}} \right) = 1$$

Definition 2. As usual under a tree graph with n vertices we will understand a function

$$\eta \colon (2, \ldots, n) \to (1, \ldots, n-1)$$

such that $\eta(i) < i$ for all $i = 2, \ldots, n$, denoting by $\sum_{\eta \colon |\eta| = n}$ the sum over all such graphs.

The graphical representation of such a function indeed resembles a tree or its branch. See for example Fig. 1.

Lemma 1. The "smoothed" finite volume distribution functions for systems of particles with many-body interaction (2.1) satisfying stability condition (2.3) can be represented in the following form

$$\rho_\Lambda(\phi_m) = \sum_{n=1}^{n_\Lambda} (-\beta)^{n-1} \sum_{2,\ldots,n}^{\Lambda} b_{\mathfrak{X}_n}(\phi_m) f_\Lambda(X_n) \tag{3.2}$$

[2] As usual in the sense that $\prod_{i=j}^{k} \equiv 1$, and $\sum_{i=j}^{k} \equiv 0$, if $j > k$.

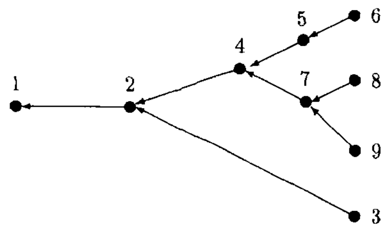

Fig. 1. A tree graph. For this tree graph the number of vertices equals 9 and $\eta(2)=1$, $\eta(3)=\eta(4)=2$, $\eta(5)=\eta(7)=4$, $\eta(6)=5$, $\eta(8)=\eta(9)=7$; $d_\eta(3)=d_\eta(6)=d_\eta(8)=d_\eta(9)=0$, $d_\eta(5)=d_\eta(1)=1$, $d_\eta(2)=d_\eta(4)=d_\eta(7)=2$.

where $n_\Lambda = |\Lambda\backslash X_1| + 1$,

$$b_{\mathfrak{X}_n}(\phi_m) = \sum_{\eta:\,|\eta|=n} \int_0^1 (ds)_{n-1}^1 \prod_{i=2}^{n} (s_{\eta(i)} \cdots s_{i-2}) \int_{S'(\Lambda)} dP_z^\Lambda(q) \langle :q^{\otimes m}:, \phi_m \rangle$$
$$\times \prod_{i=2}^{n} V_{\eta(i),i}[q] \exp[-\beta V_{X_n}(q; \mathfrak{X}_{n-1}, (s)_{n-1}^1)] \tag{3.3}$$

the values $V_X(q; \mathfrak{X}_{n-1}, (s)_{n-1}^1)$ for all X such that $X_n \subseteq X \subseteq \Lambda$ recursively given by the formula

$$V_X(q; \mathfrak{X}_i, (s)_i^1) = (1-s_i)\{V_{X_i}(q; \mathfrak{X}_{i-1}, (s)_{i-1}^1) + V_{X\backslash X_i}[q]\}$$
$$+ s_i V_X(q; \mathfrak{X}_{i-1}, (s)_{i-1}^1) \tag{3.4}$$

with $V_X(q; \mathfrak{X}_0, (s)_0^1) = V_X[q]$ by definition,

$$V_{i,j}[q] = \sum_{p=|Y_j|+1}^{M} \sum_{\substack{p_i+\cdots+p_j=p\\ p_i>0,\, p_j>0}} \sum_{\substack{p_i^1+\cdots+p_i^{|Y_i|}=p_i\\ \cdots \\ p_j^1+\cdots+p_j^{|Y_j|}=p_j\\ p_j^1,\ldots,p_j^{|Y_j|}>0}} \frac{1}{p_i^1!\cdots p_i^{|Y_i|}!\cdots p_j^1!\cdots p_j^{|Y_j|}!}$$
$$\times V_p(q; \{\Delta_i^1\}^{p_i^1}, \ldots, \{\Delta_i^{|Y_i|}\}^{p_i^{|Y_i|}}, \ldots, \{\Delta_j^1\}^{p_j^1}, \ldots, \{\Delta_j^{|Y_j|}\}^{p_j^{|Y_j|}}) \tag{3.5}$$

and

$$f_\Lambda(X) = \frac{\Xi(\Lambda\backslash X)}{\Xi(\Lambda)} \tag{3.6}$$

It is convenient to represent every summand in the expression (3.5) in the diagram form as it was done on Fig. 2 for the case when $i=4$, $j=7$,

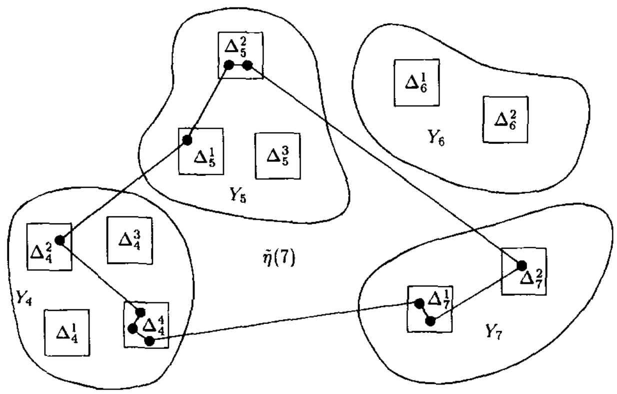

Fig. 2. "Web." On the figure pictured a summand in the expression for $V_{4,7}[q]$ (see Eq. (3.5)) for which $p_4^1 = p_4^3 = p_5^3 = p_6^1 = p_6^2 = 0$, $p_4^2 = p_5^1 = p_7^2 = 1$, $p_5^2 = p_7^1 = 2$, $p_4^4 = 3$ and therefore $p_4 = 4$, $p_5 = 3$, $p_6 = 0$, $p_7 = 3$. In constructing the "augmented" tree graph which corresponds to the usual tree graph from Fig. 1, $\tilde{\eta}(7)$ substitute the line connecting the vertices 7 and 4. See also Fig. 3.

$|Y_4| = 4$, $|Y_5| = 3$ and $|Y_6| = |Y_7| = 2$. Figure 2 also illustrates the fact that some numbers p (but not the one corresponding to the end index j) can put on the zero value.

For the following exposition it is also convenient to introduce the notion of an "augmented" tree graph which can be obtained from the usual tree graph by replacing every its rib representing $V_{i,j}[q]$ as a whole with a "web" graph representing a summand in the expression (3.5). This allows us to rewrite expression (3.3) for $b_{X_n}(\phi_m)$ in the followng form:

$$\begin{aligned} b_{\mathfrak{X}_n}(\phi_m) = &\sum_{\eta:\, |\eta| = n} \int_0^1 (ds)_{n-1}^1 \prod_{i=2}^{n} (s_{\eta(i)} \cdots s_{n-2}) \\ &\times \sum_{\tilde{\eta}(\eta,\, |Y_1|,\ldots,\, |Y_n|)} \int_{S'(\Lambda)} dP_z^\Lambda(q) \langle :q^{\otimes m}:, \phi_m \rangle \\ &\times \prod_{i=2}^{n} \frac{V_{\tilde{\eta}(i)}[q]}{p_{\tilde{\eta}(i)}!} \exp[-\beta V_{X_n}(q; \mathfrak{X}_{n-1}, (s)_{n-1}^1)] \end{aligned} \tag{3.7}$$

where

$$\sum_{\tilde{\eta}(\eta,\, |Y_1|,\ldots,\, |Y_n|)} = \prod_{i=2}^{n} \left(\sum_{p(i) = |Y_i| + 1}^{M} \sum_{\substack{p(i)_{\eta(i)} + \cdots + p(i)_i = p(i) \\ p(i)_i > 0}} \sum_{\substack{p(i)_{\eta(i)}^1 + \cdots + p(i)_{\eta(i)}^{|Y_{\eta(i)}|} = p(i)_{\eta(i)} \\ \cdots \\ p(i)_{\eta(i)}^1 + \cdots + p(i)_i^{|Y_i|} = p(i)_i \\ p(i)_i^1, \ldots, p(i)_i^{|Y_i|} > 0}} \right)$$

denotes the sum over all the possible "augmented" tree graphs corresponding to the usual tree graph η and a finite sequence $|Y_1|,\dots,|Y_n|$, $p_{\tilde{\eta}(i)}! = p(i)^1_{\eta(i)}!\cdots p^{|Y_{\eta(i)}|}_{\eta(i)}!\cdots p(i)^1_i!\cdots p^{|Y_i|}_i!$, and

$$V_{\tilde{\eta}(i)}[q] = V_{p(i)}\big(q;\ \{\Delta^1_{\eta(i)}\}^{p(i)^1_{\eta(i)}},\dots,\ \{\Delta^{|Y_{\eta(i)}|}_{\eta(i)}\}^{p(i)^{|Y_{\eta(i)}|}_{\eta(i)}},\dots,\\ \{\Delta^1_i\}^{p(i)^1_i},\dots,\ \{\Delta^{|Y_i|}_i\}^{p(i)^{|Y_i|}_i}\big)$$

A graphical representation of a summand from formula (3.7) is given on Fig. 3.

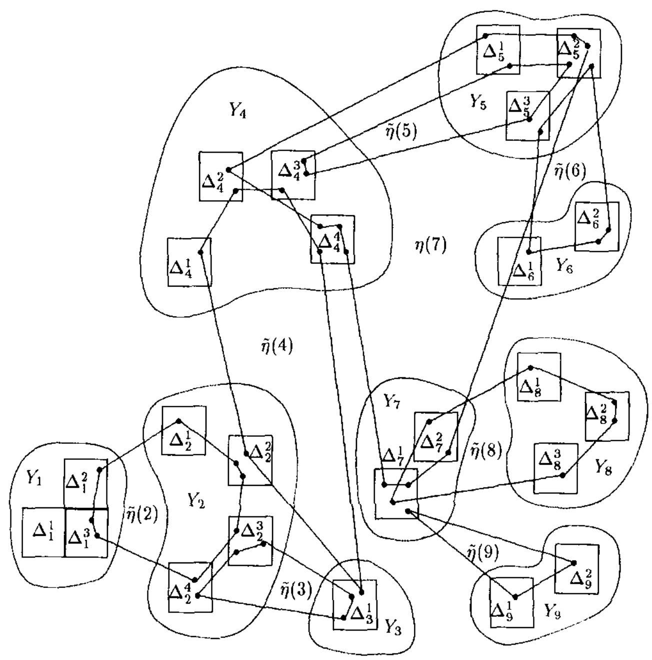

Fig. 3. An "augmented" tree graph. On the picture is given an "augmented" tree graph corresponding to the usual tree graph from Fig. 2 and the finite sequence $|Y_1|=3$, $|Y_2|=4$, $|Y_3|=1$, $|Y_4|=4$, $|Y_5|=3$, $|Y_6|=2$, $|Y_7|=2$, $|Y_8|=3$, $|Y_9|=2$. For the given graph $p(2)^1_1 = p(3)^1_2=p(3)^2_2=p(4)^1_2=p(4)^3_2=p(4)^4_2=p(5)^1_4=p(5)^2_4=p(5)^4_4=p(6)^1_5=p(7)^1_4=p(7)^3_4=p(7)^3_5=p(7)^1_6=p(7)^2_6=p(9)^2_7=p(9)^1_8=p(9)^2_8=p(9)^3_8=0$, $p(2)^2_1=p(2)^1_2=p(2)^3_2=p(2)^4_2=p(3)^4_2=p(4)^2_2=p(4)^1_3=p(4)^1_4=p(4)^2_4=p(4)^3_4=p(4)^4_4=p(5)^1_5=p(5)^2_5=p(5)^3_5=p(6)^2_5=p(6)^3_5=p(6)^1_6=p(7)^2_4=p(7)^1_5=p(7)^2_7=p(8)^1_7=p(8)^2_7=p(8)^1_8=p(8)^3_8=p(9)^1_7=p(9)^1_9=p(9)^2_9=1$, $p(2)^3_1=p(2)^2_2=p(3)^3_2=p(3)^1_3=p(5)^3_4=p(6)^2_6=p(7)^2_5=p(7)^1_7=p(8)^2_8=2$, $p(7)^4_4=3$ and therefore $p(7)_6=p(9)_8=0$, $p(4)_2=p(4)_3=p(9)_7=1$, $p(3)_3=p(5)_4=p(6)_5=p(8)_7=p(9)_9=2$, $p(2)_1=p(3)_2=p(5)_5=p(6)_6=p(7)_5=p(7)_7=3$, $p(4)_4=p(7)_4=4$, $p(2)_2=5$, $p(9)=3$, $p(3)=p(5)=p(6)=5$, $p(4)=p(8)=6$, $p(2)=8$, $p(7)=10$. For the given graph also $n^1_1=0$, $n^2_1=n^3_1=n^1_2=n^1_4=n^1_6=n^2_6=n^1_8=n^2_8=n^3_8=n^1_9=n^2_9=1$, $n^2_2=n^3_2=n^4_2=n^1_3=n^2_4=n^3_4=n^4_4=n^1_5=n^3_5=n^2_7=2$ and $n^2_5=n^1_7=3$.

Proof of Lemma 1. It can be shown by induction that $V_\Lambda(q; \mathfrak{X}_{n-1}, (s)_{n-1}^1)$, recursively given by the formula (3.4), can be explicitly written in the following form:

$$\begin{aligned}
&V_\Lambda(q; \mathfrak{X}_{n-1}, (s)_{n-1}^1)\\
&\quad = \sum_{i=1}^{n-1} V_{Y_i}[q] + V_{\Lambda \setminus X_{n-1}}[q] + \sum_{1 \leqslant i < j \leqslant n-1} s_i \cdots s_{j-1}\\
&\quad \times \sum_{p=2}^{M} \sum_{\substack{p_i + \cdots + p_j = p \\ p_i, p_j > 0}} \frac{1}{p_i! \cdots p_j!} V_p(q; \{Y_i\}^{p_i}, \ldots, \{Y_j\}^{p_j})\\
&\quad + \sum_{i=1}^{n-1} s_i \cdots s_{n-1} \sum_{p=2}^{M} \sum_{\substack{p_i + \cdots + p_n = p \\ p_i, p_n > 0}} \frac{1}{p_i! \cdots p_j!}\\
&\quad \times V_p(q; \{Y_i\}^{p_i}, \ldots, \{Y_{n-1}\}^{p_{n-1}}, \{\Lambda \setminus Y_{n-1}\}^{p_n})
\end{aligned} \tag{3.8}$$

The assertion of Lemma 1 will be proved if we show that on the kth step of the cluster expansion we have:

$$\begin{aligned}
\rho_\Lambda(\phi_m) = &\sum_{n=1}^{k} (-\beta)^{n-1} \sum_{2,\ldots,n}^{\Lambda} b_{\mathfrak{X}_n}(\phi_m) f_\Lambda(X_n)\\
&+ (-\beta)^k \Xi(\Lambda)^{-1} \sum_{2,\ldots,k+1}^{\Lambda} \sum_{\eta:\, |\eta| = k+1} \int_0^1 (ds)_k^1\\
&\times \prod_{i=2}^{k+1} (s_{\eta(i)} \cdots s_{i-2}) \int_{S'(\Lambda)} dP_{\mathrm{z}}^{\Lambda}(q) \langle :q^{\otimes m}:, \phi_m \rangle\\
&\times \prod_{i=2}^{k+1} V_{\eta(i), i}[q] \exp[-\beta V_\Lambda(q; \mathfrak{X}_k, (s)_k^1)]
\end{aligned} \tag{3.9}$$

For $k = 0$ formula (3.9) coincides with expression (2.5), and therefore it is obviously true. Let us assume that formula (3.9) is true for some k and show that then it is true also for $k+1$. Indeed, taking into account that $V_\Lambda(q; \mathfrak{X}_{k+1}, (s)_k^1, s_{k+1} = 1) = V_\Lambda(q; \mathfrak{X}_k, (s)_k^1)$ and $V_\Lambda(q; \mathfrak{X}_{k+1}, (s)_k^1, s_{k+1} = 0) = V_{X_{k+1}}(q; \mathfrak{X}_k, (s)_k^1) + V_{\Lambda \setminus X_{k+1}}[q]$, implementing the Newton-Leibnitz formula for the function $\exp[-\beta V_\Lambda(q; \mathfrak{X}_k, (s)_k)]$:

$$\begin{aligned}
&\exp[-\beta V_\Lambda(q; (s)_k^1, s_{k+1} = 1)]\\
&\quad = \exp[-\beta V_\Lambda(q; \mathfrak{X}_{k+1}, (s)_k^1, s_{k+1} = 0)]\\
&\quad\quad + (-\beta) \int_0^1 (ds)_{k+1}^1 \frac{d}{ds_{k+1}} \exp[-\beta V_\Lambda(q; \mathfrak{X}_{k+1}, (s)_{k+1}^1)]
\end{aligned}$$

on the left-hand side of (3.9) and using the "cluster property" of the Poisson measure, one can obtain that

$$\begin{aligned}
\rho_\Lambda(\phi_m) = &\sum_{n=1}^{k+1} (-\beta)^{n-1} \sum_{2,\ldots,n}^{\Lambda} b_{\mathfrak{X}_n}(\phi_m)\, f_\Lambda(X_n) \\
&+ (-\beta)^{k+1}\, \Xi(\Lambda)^{-1} \sum_{2,\ldots,k+1}^{\Lambda} \sum_{\eta:\,|\eta|=k+1} \int_0^1 (ds)_k^1 \\
&\times \prod_{i=2}^{k+1} (s_{\eta(i)}\cdots s_{i-2}) \int_{S'(\Lambda)} dP_z^\Lambda(q) \langle :q^{\otimes m}:, \phi_m\rangle \\
&\times \prod_{i=2}^{k+1} V_{\eta(i),i}[q] \left\{ \frac{d}{ds_{k+1}} V_\Lambda(q; \mathfrak{X}_{k+1}, (s)_{k+1}^1) \right\} \\
&\times \exp[-\beta V_\Lambda(q; \mathfrak{X}_{k+1}, (s)_{k+1}^1)]
\end{aligned} \tag{3.10}$$

From formula (3.8) it follows that

$$\begin{aligned}
&\frac{d}{ds_{k+1}} V_\Lambda(q; \mathfrak{X}_{k+1}, (s)_{k+1}^1) \\
&\quad = \sum_{i=1}^{k+1} (s_i \cdots s_k) \sum_{p=2}^{M} \sum_{\substack{p_i+\cdots+p_{k+2}=p \\ p_i,\, p_{k+2}>0}} \frac{1}{p_i!\cdots p_{k+2}!} \\
&\quad \times V_p(q; \{Y_i\}^{p_i}, \ldots, \{Y_{k+1}\}^{p_{k+1}}, \{\Lambda \backslash X_{k+1}\}^{p_{k+2}})
\end{aligned}$$

and therefore, making the following transformation

$$\begin{aligned}
&\sum_{p=2}^{M} \sum_{\substack{p_i+\cdots+p_{k+2}=p \\ p_i,\, p_{k+2}>0}} \frac{1}{p_i!\cdots p_{k+2}!} V_p(q; \{Y_i\}^{p_i}, \ldots, \{Y_{k+1}\}^{p_{k+1}}, \{\Lambda \backslash X_{k+1}\}^{p_{k+2}}) \\
&\quad = \sum_{p=2}^{M} \sum_{\substack{p_i+\cdots+p_{k+2}=p \\ p_i,\, p_{k+2}>0}} \sum_{\substack{p_i^1+\cdots+p_i^{|Y_i|}=p_i \\ \cdots \\ p_{k+2}^1+\cdots+p_{k+2}^{|\Lambda\backslash X_{k+1}|}=p_{k+2}}} \\
&\quad \times \frac{1}{p_i^1!\cdots p_i^{|Y_i|}!\cdots p_{k+2}^1!\cdots p_{k+2}^{|\Lambda\backslash X_{k+1}|}!} \\
&\quad \times V_p(q; \{\Delta_i^1\}^{p_i^1}, \ldots, \{\Delta_i^{|Y_i|}\}^{p_i^{|Y_i|}}, \{\Delta_{k+2}^1\}^{p_{k+2}^1}, \ldots, \{\Delta_{k+2}^{|\Lambda\backslash X_{k+1}|}\}^{p_{k+2}^{|\Lambda\backslash X_{k+1}|}})
\end{aligned}$$

$$
\begin{aligned}
&= \sum_{p=2}^{M} \sum_{\substack{p_i + \cdots + p_{k+2} = p \\ p_i, p_{k+2} > 0}} \sum_{|Y_{k+2}|=2}^{p_{k+2}} \sum_{\substack{\Delta_{k+2}^1, \ldots, \Delta_{k+2}^{|Y_k|} \subset \Lambda \backslash X_{k+1} \\ \Delta_{k+2}^r \neq \Delta_{k+2}^l,\, r \neq l}} \frac{1}{|Y_{k+2}|!} \\
&\quad \times \sum_{\substack{p_i^1 + \cdots + p_i^{|Y_i|} = p_i \\ \cdots \\ p_{k+2}^1 + \cdots + p_{k+2}^{|Y_{k+2}|} = p_{k+2} \\ p_{k+2}^1, \ldots, p_{k+2}^{|Y_{k+2}|} > 0}} \frac{1}{p_i^1! \cdots p_i^{|Y_i|}! \cdots p_{k+2}^1! \cdots p_{k+2}^{|Y_{k+2}|}!} \\
&\quad \times V_p(q; \{\Delta_i^1\}^{p_i^1}, \ldots, \{\Delta_i^{|Y_i|}\}^{p_i^{|Y_i|}}, \{\Delta_{k+2}^1\}^{p_{k+2}^1}, \ldots, \{\Delta_{k+2}^{|\Lambda \backslash X_{k+1}|}\}^{p_{k+2}^{|\Lambda \backslash X_{k+1}|}}) \\
&= \sum_{Y_{k+2} \subset \Lambda \backslash X_{k+1}} \sum_{p=|Y_{k+2}|+1}^{M} \sum_{\substack{p_i + \cdots + p_{k+2} = p \\ p_i, p_{k+2} > 0}} \sum_{|Y_{k+2}|=2}^{p_{k+2}} \\
&\quad \times \sum_{\substack{\Delta_{k+2}^1, \ldots, \Delta_{k+2}^{|Y_k|} \subset \Lambda \backslash X_{k+1} \\ \Delta_{k+2}^r \neq \Delta_{k+2}^l,\, r \neq l}} \frac{1}{|Y_{k+2}|!} \\
&\quad \times \sum_{\substack{p_i^1 + \cdots + p_i^{|Y_i|} = p_i \\ \cdots \\ p_{k+2}^1 + \cdots + p_{k+2}^{|Y_{k+2}|} = p_{k+2} \\ p_{k+2}^1, \ldots, p_{k+2}^{|Y_{k+2}|} > 0}} \frac{1}{p_i^1! \cdots p_i^{|Y_i|}! \cdots p_{k+2}^1! \cdots p_{k+2}^{|Y_{k+2}|}!} \\
&\quad \times V_p(q; \{\Delta_i^1\}^{p_i^1}, \ldots, \{\Delta_i^{|Y_i|}\}^{p_i^{|Y_i|}}, \{\Delta_{k+2}^1\}^{p_{k+2}^1}, \ldots, \{\Delta_{k+2}^{|\Lambda \backslash X_{k+1}|}\}^{p_{k+2}^{|\Lambda \backslash X_{k+1}|}})
\end{aligned}
$$

we can rewrite (3.10) as follows

$$
\begin{aligned}
\rho_\Lambda(\phi_m) &= \sum_{n=1}^{k+1} (-\beta)^{n-1} \sum_{2, \ldots, n}^{\Lambda} b_{\mathfrak{X}_n}(\phi_m) f_\Lambda(X_n) \\
&\quad + (-\beta)^{k+1} \Xi(\Lambda)^{-1} \sum_{2, \ldots, k+2}^{\Lambda} \sum_{\eta: |\eta| = k+2} \int_0^1 (ds)_{k+1}^1 \\
&\quad \times \prod_{i=2}^{k+2} (s_{\eta(i)} \cdots s_{i-2}) \int_{S'(\Lambda)} dP_z^\Lambda(q) \langle :q^{\otimes m}:, \phi_m \rangle \\
&\quad \times \prod_{i=2}^{k+2} V_{\eta(i), i}[q] \exp[-\beta V_\Lambda(q; \mathfrak{X}_{k+1}, (s)_k^1)]
\end{aligned}
$$

which is exactly the formula (3.9) in which k is replaced by $k+1$. ∎

4. MAIN RESULTS

Definition 3. Let $\operatorname{dist}(\Delta, \Delta')$ denote the distance between the centers of cubes Δ and Δ'. We will say that the many-body potential (2.1) exponentially decreases on infinity "in the integral sense" if for some $\alpha > 0$

$$\tilde{v} = \max_{\Delta \in \mathfrak{T}_{\mathbb{R}^3}} \sum_{p=2}^{M} \sum_{\Delta^{(1)},\ldots,\Delta^{(p-1)} \in \mathfrak{T}_{\mathbb{R}^3}} \tilde{V}(\Delta, \Delta^{(1)},\ldots, \Delta^{p-1})\, e^{\alpha \operatorname{diam}\{\Delta, \Delta^{(1)},\ldots,\Delta^{(p-1)}\}} < \infty \tag{4.1}$$

where

$$\tilde{V}(\Delta_1,\ldots,\Delta_p) = \max_{1 \leqslant l \leqslant p-1} \max_{\pi \in \mathfrak{S}_p} \sup_{\substack{x_{\pi(k)} \in \Delta_{\pi(k)} \\ \forall k = 1,\ldots, l}} \left(\int_{\Delta_{\pi(l+1)}} dx_{\pi(l+1)} \cdots \right.$$

$$\left. \times \int_{\Delta_{\pi(p)}} dx_{\pi(p)}\, V_p^2(x_1,\ldots,x_p) \right)^{1/2}$$

$$\operatorname{diam}\{\Delta_1,\ldots,\Delta_p\} = \max_{1 \leqslant i < j \leqslant p} \operatorname{dist}(\Delta_i, \Delta_j)$$

and $\mathfrak{S}_p$ the group of permutations of $\{1,\ldots,p\}$.

Theorem 3. For sufficiently small β the thermodynamic limit for the distribution functions of a classical system of particles interacting through the stable[3] many-body potential (2.1) which exponentially decreases on infinity "in the integral sense" exists and can be represented in the form of the following absolutely convergent series

$$\rho(\phi_m) = \sum_{n=1}^{\infty} (-\beta)^{n-1} \sum_{2,\ldots,n}^{\mathbb{R}^3} b_{\mathfrak{X}_n}(\phi_m)\, f(X_n) \tag{4.2}$$

where $b_{\mathfrak{X}_n}(\phi_m)$ is given by formula (3.3) and

$$f(X) = \lim_{\Lambda \nearrow \mathbb{R}^3} f_\Lambda(X) \tag{4.3}$$

Theorem 4. With the same conditions as in Theorem 3 and with $\phi'_k \in \mathscr{D}(\Lambda^k)$, $\phi''_l \in \mathscr{D}(\Lambda^l)$ supported in $\{X'_1\}^k$ and $\{X''_1\}^l$ respectively, there

[3] I.e., satisfying the stability condition (2.3).

exists an independent of Λ and $\operatorname{dist}(X_0', X_0'')$ constant $C_2 = C_2(A, B, \beta, z)$ such that for the same α as in (4.1)

$$|\rho_\Lambda(\phi_k' \otimes \phi_l'') - \rho_\Lambda(\phi_k')\,\rho_\Lambda(\phi_l'')| \leqslant C_2 \exp\left[-\frac{\alpha}{2}\operatorname{dist}(X_1', X_2'')\right] \quad (4.4)$$

5. THERMODYNAMIC LIMIT. PROOF OF CONVERGENCE

Before proving Theorem 3 we need to show some auxiliary propositions.

Lemma 2. There exists a constant C_0 independent of the volume Λ, inverse temperature β and "augmented" tree graph $\tilde{\eta}$ such that

$$\left(\int_{S'(\Lambda)} dP_z^\Lambda(q) \prod_{i=2}^{n} V_{\tilde{\eta}(i)}^2[q]\right)^{1/2} \leqslant C_0^{n-1} \prod_{i=2}^{n} (\tilde{V}_{\tilde{\eta}(i)} \exp[\alpha \operatorname{diam}\{\tilde{\eta}(i)\}]) \quad (5.1)$$

where $\tilde{V}_{\tilde{\eta}(i)} = \tilde{V}(\Delta_{\eta(i)}^1, \dots, \Delta_{\eta(i)}^{|Y_{\eta(i)}|}, \dots, \Delta_i^1, \dots, \Delta_i^{|Y_i|})$ and $\operatorname{diam}\{\tilde{\eta}(i)\} = \operatorname{diam}\{\Delta_{\eta(i)}^1, \dots, \Delta_{\eta(i)}^{|Y_{\eta(i)}|}, \dots, \Delta_i^1, \dots, \Delta_i^{|Y_i|}\}$.

Proof. The left-hand side of inequality (5.1) can be graphically represented by a "magnified[4] augmented tree graph" $\tilde{\eta}'$ obtained from the usual "augmented" tree graph by replacing its every polygon component with two ones. Using the "cluster property" of the Poisson measure together with the generalized Wick's theorem for Poisson fields and formula (1.7), one can perform an integration with respect to the Poisson measure on the left-hand side of inequality (5.1). The expression obtained after such a procedure can be described as a square root from the sum of all "magnified augmented tree graphs with pairings" corresponding to a given "augmented" tree graph $\tilde{\eta}$ (see Fig. 4).

As follows from Corollary 1, the number of such summands does not exceed $\prod_{\Delta \in \mathfrak{D}_{\tilde{\eta}}} (2p_\Delta^{\tilde{\eta}})!$, where $\mathfrak{D}_{\tilde{\eta}}$ is a set of cubes connected with a given "augmented" tree graph $\tilde{\eta}$ and $p_\Delta^{\tilde{\eta}}$ the number of points in the cube Δ. Using in every such summand over "magnified augmented graphs with pairings" the Schwarz inequality to the integral (integrals) with respect to a coordinate (coordinates) of points[5] located in the cube Δ_n^1 and taking supremum over possible allocations of the other points of the nth polygon components of it, we gain the possibility to execute such a procedure for the $(n-1)$th components and so on.

[4] We derive the term "magnified" from the fact that the graphical representation of a "magnified augmented tree graph" looks like a such many times magnified one of the usual "augmented" tree graph that becomes visible the internal structure of all its polygon components which now are doubled.

[5] The only possible cases are one or two coordinates in Δ_n^1.

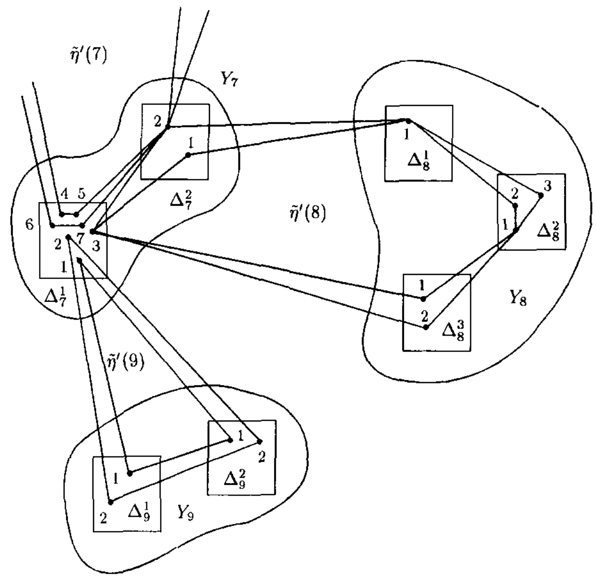

Fig. 4. A "magnified augmented tree graph with pairings." On the picture is given a part of one of the possible "magnified augmented tree graphs with pairings" corresponding to the usual "augmented" tree graph from Fig. 3.

As a result, taking into account Definition 3, we obtain that

$$\left(\int_{S'(\Lambda)} dP_z^\Lambda(q) \prod_{i=2}^n V_{\tilde{\eta}(i)}^2[q]\right)^{1/2} \leqslant \begin{Bmatrix} z^{n/2}, & z<1 \\ z^{M(n-1)}, & z>1 \end{Bmatrix} \left(\prod_{\Delta \in \mathfrak{D}_{\tilde{\eta}}} (2p_\Delta^{\tilde{\eta}})!\right)^{1/2} \prod_{i=2}^n \tilde{V}_{\tilde{\eta}(i)}$$

where the factor $z^{n/2}$ if $z<1$ or $z^{M(n-1)}$ if $z>1$, appears because of the fact that the minimal number of points in a "magnified augmented tree graph" (when for all $i=1,\dots,n$ $|Y_i|=1$ and all the possible pairings occurred) and the maximal number (when for all $i=2,\dots,n$ $p(i)=M$ and there was not any pairing) equals n and $2M(n-1)$, respectively.

The proof of Lemma 2 will be completed if we show that for some constant $\hat{C}$ independent of Λ and η

$$\left(\prod_{\Delta \in \mathfrak{D}_{\tilde{\eta}}} (2p_\Delta^{\tilde{\eta}})!\right)^{1/2} \prod_{i=2}^n \exp[-\alpha \operatorname{diam}\{\tilde{\eta}(i)\}] \leqslant \hat{C}^{M(n-1)} \tag{5.2}$$

To do so, let us rewrite the left-hand side of (5.2) in the following form

$$\prod_{\Delta \in \mathfrak{D}_{\tilde{\eta}}} \left(\sqrt{(2p_\Delta^{\tilde{\eta}})!}\, \exp\left[-\alpha \sum_{i=2}^n \frac{p_\Delta^{\tilde{\eta}}(i)}{p^{\tilde{\eta}}(i)} \operatorname{diam}\{\tilde{\eta}(i)\}\right]\right)$$

where $p_\Lambda^{\tilde\eta}(i)$ is the number of points from the ith component of the "augmented" tree graph $\tilde\eta$ in the cube Δ and $p^{\tilde\eta}(i) = \sum_{\Lambda \in \mathfrak{D}_{\tilde\eta}} p_\Lambda^{\tilde\eta}(i)$. Using the facts that $|\mathfrak{D}_{\tilde\eta}| \leqslant 1 + (M-1)(n-1) \leqslant M(n-1)$,

$$\prod_{\Lambda \in \mathfrak{D}_{\tilde\eta}} \left(\sqrt{(2p_\Lambda^{\tilde\eta})!} \exp\left[-\alpha \sum_{i=2}^{n} \frac{p_\Lambda^{\tilde\eta}(i)}{p^{\tilde\eta}(i)} \operatorname{diam}\{\tilde\eta(i)\} \right] \right)$$
$$\leqslant \left(\max_{\Lambda, \tilde\eta} \left\{ \sqrt{(2p_\Lambda^{\tilde\eta})!} \exp\left[-\alpha \sum_{i=2}^{n} \frac{p_\Lambda^{\tilde\eta}(i)}{p^{\tilde\eta}(i)} \operatorname{diam}\{\tilde\eta(i)\} \right] \right\} \right)^{|\mathfrak{D}_{\tilde\eta}|}$$

and

$$\sqrt{(2p_\Lambda^{\tilde\eta})!} \exp\left[-\alpha \sum_{i=2}^{n} \frac{p_\Lambda^{\tilde\eta}(i)}{p^{\tilde\eta}(i)} \operatorname{diam}\{\tilde\eta(i)\} \right] \to 0, \qquad n_\Lambda^{\tilde\eta} \to \infty \tag{5.3}$$

uniformly with respect to Λ and $\tilde\eta$ (see Appendix), where $n_\Lambda^{\tilde\eta}$ is the number of components of the "augmented" tree graph $\tilde\eta$ which enter the cube Δ (see Fig. 3), one can conclude that there exists such an independent from Λ and $\tilde\eta$ constant $\hat C$ that

$$\sqrt{(2p_\Lambda^{(\tilde\eta)})!} \exp\left[-\alpha \sum_{i=2}^{n} \frac{p_\Lambda^{(\tilde\eta)}(i)}{p^{(\tilde\eta)}(i)} d_{\tilde\eta(i)} \right] \leqslant \hat C \quad \blacksquare$$

Lemma 3. For the many-body potential (2.1) satisfying the stability condition (2.3) the following inequality holds:

$$\left(\int_{S'(\Lambda)} dP_z^\Lambda(q) \exp[-2\beta V_{X_n}(q; \mathfrak{X}_{n-1}, (s)_{n-1}^1)] \right)^{1/4}$$
$$\leqslant C_1(\beta)^{|X_1| + (M-1)(n-1)} \tag{5.4}$$

where $C_1(\beta) = \exp[z(e^{4\beta B} - 1)/4]$.

Proof. In terms of the Poisson field the stability condition (2.3) can be reformulated as follows: there exists such a non-negative constant B that for any measurable set $X \subset \Lambda$ the inequality $V_X[q] \geqslant -B \int_X q(x)\, dx$ holds almost everywhere with respect to the measure $P_z^\Lambda(\cdot)$. From this and formula (3.4) by induction (see ref. 5 for details), it follows that $V_{X_n}(q; \mathfrak{X}_i, (s)_i^1) \geqslant -B \int_{X_n} q(x)\, dx$ also holds almost everywhere with respect to $P_z^\Lambda(\cdot)$. Finally, using the latter formula together with (1.4), (1.2) and the inequality $|X_n| \leqslant |X_1| + (M-1)(n-1)$, one can obtain that

$$\int_{S'(\Lambda)} dP_z^\Lambda(q) \exp[-4\beta V(q, X_n, (s)_{n-1}^1)]$$
$$\leqslant \int_{S'(\Lambda)} dP_z^\Lambda(q) \exp\left[4\beta B \int_{X_n} q(x)\, dx\right]$$
$$= \int_{S'(X_n)} dP_z^{X_n} \exp\left[4\beta B \int_{X_n} q(x)\, dx\right]$$
$$= e^{-z|X_n|} \sum_{k=0}^{\infty} \frac{z^k}{k!} \int_{(X_n)^k} (dx)_k^1 \exp[4\beta Bk]$$
$$= e^{-z|X_n|} \sum_{k=0}^{\infty} \frac{z^k}{k!} (|X_n| \exp[4\beta B])^k$$
$$= \exp[z\,|X_n|(e^{4\beta B} - 1)]$$
$$= \exp[z(|X_1| + (n-1))(e^{4\beta B} - 1)] \quad ▮$$

Lemma 4. If the potential energy (2.2) satisfies the stability condition (2.3) then for any finite[6] X and Λ such that $X \subseteq \Lambda \subset \mathbb{R}^3$ the inequality $|f_\Lambda(X)| \leqslant e^{z|X|}$ holds.

Proof. From formulae (3.6), (2.7) and the definition of the grand partition function it follows that

$$f_\Lambda(X) = \frac{e^{-z|\Lambda\setminus X|} \sum_{n=0}^{\infty} z^n/n! \int_{(\Lambda\setminus X)^n} (dx)_n^1 \exp[-\beta V(x)_n^1]}{e^{-z|\Lambda|} \sum_{n=0}^{\infty} z^n/n! \int_{\Lambda^n} (dx)_n^1 \exp[-\beta V(x)_n^1]} \tag{5.5}$$

Taking into account that enlarging of the integration domain in the case when a function under integration is positive may lead only to increasing of the expression:

$$\int_{(\Lambda\setminus X)^n} (dx)_n^1 \exp[-\beta V(x)_n^1] \leqslant \int_{\Lambda^n} (dx)_n^1 \exp[-\beta V(x)_n^1]$$

we immediately obtain from (5.5) that $f_\Lambda(X) \leqslant e^{-z|\Lambda\setminus X|}/e^{-z|\Lambda|} = e^{-z|X|}$. ▮

Lemma 5. If there exist such continuous[7] functions $K_\phi(\beta)$ and $K(\beta)$ that

$$\sum_{2,\ldots,n}^{\mathbb{R}^3} b_{\mathfrak{X}_n}(\phi_m) \leqslant K_\phi(\beta)(K(\beta))^n \tag{5.6}$$

[6] I.e., such that $|X| < \infty$.
[7] In the neighborhood of zero.

then for any finite $X \subset \mathbb{R}^3$ there exists the following limit:

$$f(X) = \lim_{\Lambda \nearrow \mathbb{R}^3} f_\Lambda(X) \tag{5.7}$$

The proof can be done by using the well-known technique of equations of the Kirkwood-Salsburg type, see refs. 2, 27, and 28 for example.

Proof of Theorem 3. As follows from Lemmas 4 and 5, for proving the theorem it is sufficient to show that estimate (5.6) holds for sufficiently small β with some continuous $K_\phi(\beta)$ and $K(\beta)$. Doubly using the Schwarz inequality to the integral with respect to the Poisson measure in the expression (3.7) for $b_{\mathfrak{X}_n}(\phi_m)$ we get

$$\begin{aligned}
\sum_{2,\dots,n}^{\mathbb{R}^3} |b_{\mathfrak{X}_n}(\phi_m)| \leqslant & \sum_{2,\dots,n}^{\mathbb{R}^3} \sum_{\eta:\,|\eta|=n} \int_0^1 (ds)_{n-1}^1 \prod_{i=2}^n (s_{\eta(i)} \cdots s_{i-2}) \\
& \times \sum_{\tilde{\eta}(\eta,\,|Y_1|,\dots,\,|Y_n|)} \prod_{i=2}^n \frac{1}{p_{\tilde{\eta}(i)}!} \\
& \times \left(\int_{S'(\Lambda)} dP_z^\Lambda(q) \langle :q^{\otimes m}:, \phi_m \rangle^4 \right)^{1/4} \\
& \times \left(\int_{S'(\Lambda)} dP_z^\Lambda(q) \prod_{i=2}^n V_{\tilde{\eta}(i)}^2[q] \right)^{1/2} \\
& \times \left(\int_{S'(\Lambda)} dP_z^\Lambda(q) \exp[-2\beta V_{X_n}(q; (s)_{n-1}^1)] \right)^{1/4}
\end{aligned} \tag{5.8}$$

Making use of estimates (5.1) and (5.4), obtained in Lemmas 2 and 3, and taking into account that, as it follows from formula (1.4),

$$\begin{aligned}
& \left(\int_{S'(\Lambda)} dP_z^\Lambda(q) \langle :q^{\otimes m}:, \phi_m \rangle^4 \right)^{1/4} \\
& \quad = \left(\int_{S'(X_1)} dP_z^{X_1}(q) \langle :q^{\otimes m}:, \phi_m \rangle^4 \right)^{1/4} = C_\phi
\end{aligned}$$

is a constant dependent only from the function ϕ_m, we can rewrite (5.8) in the following form:

$$\sum_{2,\ldots,n}^{\mathbb{R}^3} |b_{\mathfrak{x}_n}(\phi_m)| \leqslant C_\phi C_1(\beta)^{|X_1|} (C_0 C_1(\beta))^{n-1}$$

$$\times \sum_{\eta:\, |\eta| = n} \int_0^1 (ds)_{n-1}^1 \prod_{i=2}^{n} (s_{\eta(i)} \cdots s_{i-2})$$

$$\times \sum_{2,\ldots,n}^{\mathbb{R}^3} \sum_{\tilde{\eta}(\eta,\, |Y_1|,\ldots,\, |Y_n|)} \prod_{i=2}^{n} \frac{\tilde{V}_{\tilde{\eta}(i)} \exp[\operatorname{diam}\{\tilde{\eta}(i)\}]}{p_{\tilde{\eta}(i)}!} \tag{5.9}$$

Using (3.1) and denoting $K_\phi = C_\phi C_1(\beta)^{|X_1|}$ and $C_0'(\beta) = C_0 C_1(\beta)$, one can rewrite (5.9) as follows:

$$\sum_{2,\ldots,n}^{\mathbb{R}^3} |b_{\mathfrak{x}_n}(\phi_m)| \leqslant K_\phi(\beta)\, C_1'(\beta)^{n-1} \sum_{\eta:\, |\eta| = n} \int_0^1 (ds)_{n-1}^1 \prod_{i=2}^{n} (s_{\eta(i)} \cdots s_{i-2})$$

$$\times \sum_{|Y_1|,\ldots,\, |Y_n| = 1}^{M-1} \sum_{\tilde{\eta}(\eta,\, |Y_1|,\ldots,\, |Y_n|)} \sum_{\substack{\Delta_2^1,\ldots,\, \Delta_2^{|Y_2|} \subset \Lambda \setminus X_1 \\ \cdots \\ \Delta_n^1,\ldots,\, \Delta_n^{|Y_n|} \subset \Lambda \setminus X_{n-1} \\ \Delta_i^k \neq \Delta_i^l,\, k \neq l}}$$

$$\times \prod_{i=2}^{n} \frac{\tilde{V}_{\tilde{\eta}(i)} \exp[\operatorname{diam}\{\tilde{\eta}(i)\}]}{p_{\tilde{\eta}(i)}!\, |Y_i|!} \tag{5.10}$$

Defining "simple" augmented tree graphs as such augmented tree graphs for which $p(i) = |Y_i| + 1$, $\forall i = 2,\ldots, n$, we can consider an arbitrary augmented tree graph as a particular case of the corresponding "simple" augmented tree graph when some cubes $\Delta_1^1,\ldots, \Delta_n^{|Y_n|}$ coincide, see Fig. 5.

So, omitting in formula (5.10) the division by $p_{\tilde{\eta}(i)}!\, |Y_i|!$, we can, not violating the inequality, extend summation over cubes $\Delta_2^1,\ldots, \Delta_n^{|Y_n|}$, in such a way that some of them may coincide, restricting at the same time summation over augmented tree graphs $\tilde{\eta}$ only by the "simple" ones:

$$\sum_{2,\ldots,n}^{\mathbb{R}^3} |b_{\mathfrak{x}_n}(\phi_m)| \leqslant K_\phi(\beta)\, C_1'(\beta)^{n-1} \sum_{\eta:\, |\eta| = n} \int_0^1 (ds)_{n-1}^1 \prod_{i=2}^{n} (s_{\eta(i)} \cdots s_{i-2})$$

$$\times \sum_{|Y_1|,\ldots,\, |Y_n| = 1}^{M-1} \sum_{\tilde{\eta}(\eta,\, |Y_1|,\ldots,\, |Y_n|)}^{\text{simple}} \sum_{\substack{\Delta_2^1 \subset \Lambda \setminus X_1,\ldots,\, \Delta_n^1 \subset \Lambda \setminus X_{n-1} \\ \Delta_2^2,\ldots,\, \Delta_2^{|Y_2|} \subset \Lambda \\ \cdots \\ \Delta_n^2,\ldots,\, \Delta_n^{|Y_n|} \subset \Lambda \setminus X_{\eta(n)-1}}}$$

$$\times \prod_{i=2}^{n} \tilde{V}_{\tilde{\eta}(i)} \exp[\operatorname{diam}\{\tilde{\eta}(i)\}]. \tag{5.11}$$

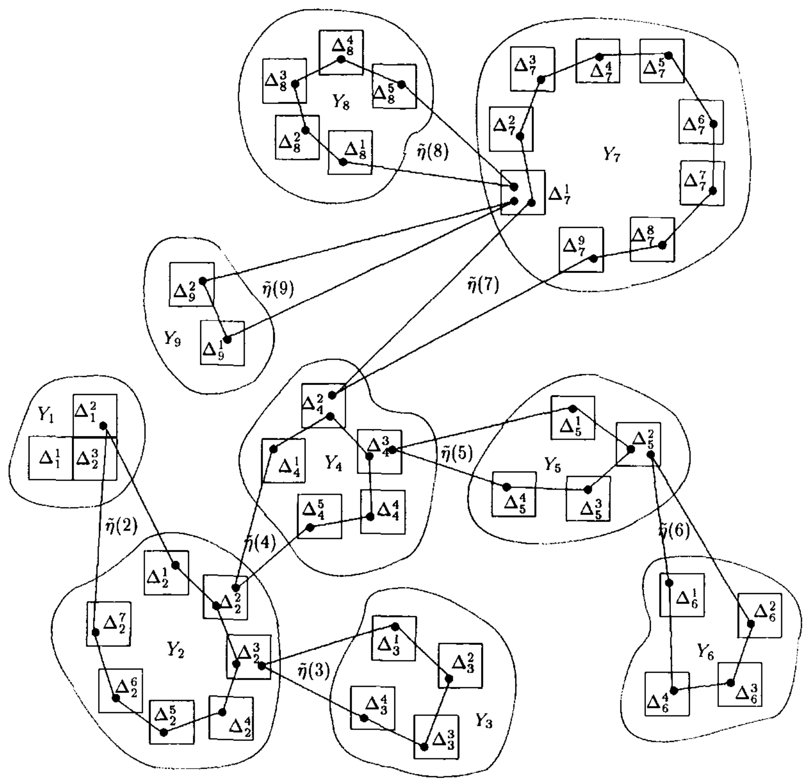

Fig. 5. A "simple" augmented tree graph. The augmented tree graph from Fig. 3 can be obtained from the given "simple" augmented tree graph for example when the cubes Δ_2^6, Δ_2^7 coincide with the cube Δ_1^3, the cube Δ_2^5 with the cube Δ_2^2, Δ_3^4 with Δ_2^3, Δ_3^3 with Δ_2^4, Δ_3^2 with Δ_3^1, Δ_4^5 with Δ_3^1, Δ_5^4 with Δ_4^3, Δ_6^4 with Δ_5^3, Δ_6^3 with Δ_6^2, the cubes Δ_7^9, Δ_7^8, Δ_7^7 with Δ_4^4, the cube Δ_7^6 with Δ_5^1, the cubes Δ_7^5, Δ_7^4 with Δ_5^2, the cube Δ_7^3 with Δ_7^1, Δ_8^5 with Δ_7^2, and Δ_8^4 with Δ_8^2. The cube Δ_7^1 is the "initial" for the 9th and 8th component of the graph, the cube Δ_4^2 is the "initial" for the 7th component, the cube Δ_5^2 for the 6th component, Δ_4^3 for the 5th one, Δ_2^2 for the 4th, Δ_2^3 for the 3rd, and Δ_1^2 for the 2nd.

It should be noted that, although after such resummation some summands from the right-hand side of formula (5.10) will be counted more then one time due to the fact that some of the "augmented" tree graphs can be obtained from different "simple" augmented tree graphs and in a different way from the same "simple" augmented tree graph, no one of them can be lost.

Consequently executing summation with respect to $\Delta_i^1, \ldots, \Delta_i^{|Y_i|}$ and taking maximum[8] with respect to the "initial" (see Fig. 5 for explanation of

[8] This is not necessarily for translationally invariant potential.

this term) cube of the ith component of the "simple" augmented tree graph for $i = n,\dots, 2$, we obtain after $n-1$ steps that

$$\sum_{\substack{\Delta_2^1 \subset \Lambda\backslash X_1,\dots, \Delta_n^1 \subset \Lambda\backslash X_{n-1} \\ \Delta_2^2,\dots,\Delta_2^{|Y_2|} \subset \Lambda \\ \cdots \\ \Delta_n^2,\dots,\Delta_n^{|Y_n|} \subset \Lambda\backslash X_{\eta(n)-1}}} \prod_{i=2}^{n} \tilde{V}_{\tilde{\eta}(i)} \exp[\operatorname{diam}\{\tilde{\eta}(i)\}]$$
$$\leqslant \prod_{i=2}^{n} \left(\max_{\Delta \in \mathfrak{D}_{\mathbb{R}^3}} \sum_{\Delta^1,\dots,\Delta^{|Y_n|} \in \mathfrak{D}_{\mathbb{R}^3}} \tilde{V}(\Delta, \Delta^1,\dots, \Delta^{|Y_n|}) \right.$$
$$\left. \times \exp[\alpha \operatorname{diam}\{\Delta, \Delta^1,\dots, \Delta^{|Y_n|}\}] \right) \tag{5.12}$$

Substituting (5.12) in (5.11) and taking into account that

$$\sum_{\tilde{\eta}(\eta, |Y_1|,\dots,|Y_n|)}^{\text{simple}} 1 \leqslant (\max\{|X_1|, (M-1)\})^{n-1}$$

we come to

$$\sum_{2,\dots,n}^{\mathbb{R}^3} |b_{\mathfrak{X}_n}(\phi_m)|$$
$$\leqslant K_\phi(\beta)\, K'(\beta)^{n-1} \sum_{\eta:\, |\eta|=n} \int_0^1 (ds)_{n-1}^1 \prod_{i=2}^{n} (s_{\eta(i)} \cdots s_{i-2})$$
$$\times \sum_{|Y_1|,\dots,|Y_n|=1}^{M-1} \prod_{i=2}^{n} \left(\max_{\Delta \in \mathfrak{D}_{\mathbb{R}^3}} \sum_{\Delta^1,\dots,\Delta^{|Y_n|} \in \mathfrak{D}_{\mathbb{R}^3}} \tilde{V}(\Delta, \Delta^1,\dots, \Delta^{|Y_n|}) \right.$$
$$\left. \times \exp[\alpha \operatorname{diam}\{\Delta, \Delta^1,\dots, \Delta^{|Y_n|}\}] \right) \tag{5.13}$$

where $K'(\beta) = \max\{|X_1|, (M-1)\}\, C_1'(\beta)$. Finally, summing over $|Y_2|,\dots, |Y_n|$ and using the following estimate[5]

$$\sum_{\eta = \eta(\{Y\}_n^1)} \int_0^1 (ds)_{n-1}\, f_\eta(s)_{n-2} \leqslant e^{n-1} \tag{5.14}$$

we get that $\sum_{2,\dots,n}^{\mathbb{R}^3} |b_{\mathfrak{X}_n}(\phi_m)| \leqslant K_\phi(\beta)\, K(\beta)^{n-1}$ with $K(\beta) = e\tilde{v}K'(\beta)$. ∎

The proof of Theorem 4 is a sequence of the convergence of the cluster expansions, i.e., Theorem 3, see refs. 29, 9 for example.

We conclude our paper with a remark that the same results with little changes in proofs may be obtained for the many-component charged particles systems with many-body interaction similar to the one given by formula (2.1).

APPENDIX

Here we are going to show that formula (5.3), used in the proof of Lemma 2, holds uniformly in Λ and $\tilde{\eta}$. First of all, let us renumerate all the components of a given augmented tree graph $\tilde{\eta}$ entering an arbitrary fixed cube Δ (i.e., for which $p_\Lambda^{\tilde{\eta}}(i) \neq 0$) in the order of increasing of their diameters $d_{\tilde{\eta}(i)}$, i.e., construct such a finite sequence $i_1,..., i_{n_\Lambda^{\tilde{\eta}}}$ that for all k and l such that $1 \leqslant k < l \leqslant n_\Lambda^{\tilde{\eta}}$, $\operatorname{diam}\{\tilde{\eta}(i_k)\} \leqslant \operatorname{diam}\{\tilde{\eta}(i_l)\}$ and $p_\Lambda^{\tilde{\eta}}(i_k) \neq 0$, $p_\Lambda^{\tilde{\eta}}(i_l) \neq 0$.

It is easy to see (for example, by putting the cube Δ in the center of an imaginary cube with the rib's length $(2 \operatorname{diam}\{\tilde{\eta}(i_k)\} + 1)$ and counting the maximal number of different graphs enclosed in it and entering Δ) that even for the most compact graph

$$\begin{aligned} k &\leqslant \sum_{p=2}^{M} (2 \operatorname{diam}\{\tilde{\eta}(i_k)\} + 1)^{3(p-1)} \\ &\leqslant (M-1)(2 \operatorname{diam}\{\tilde{\eta}(i_k)\} + 1)^{3(M-1)} \end{aligned} \tag{A.1}$$

Inverting the inequality (A.1), we obtain that $\operatorname{diam}\{\tilde{\eta}(i_k)\} \geqslant \frac{1}{2}(k/(M-1))^{1/[3(M-1)]}$ and therefore

$$\begin{aligned} \sum_{i=2}^{n} \frac{p_\Lambda^{\tilde{\eta}}(i)}{p^{\tilde{\eta}}(i)} \operatorname{diam}\{\tilde{\eta}(i)\} &= \sum_{k=1}^{n_\Lambda^{\tilde{\eta}}} \frac{p_\Lambda^{\tilde{\eta}}(i_k)}{p^{\tilde{\eta}}(i_k)} \operatorname{diam}\{\tilde{\eta}(i_k)\} \\ &\geqslant \frac{1}{M} \sum_{k=1}^{n_\Lambda^{\tilde{\eta}}} \operatorname{diam}\{\tilde{\eta}(i_k)\} \\ &\geqslant \frac{1}{2M} \sum_{k=1}^{n_\Lambda^{\tilde{\eta}}} \left(\frac{k}{M-1}\right)^{1/[3(M-1)]} \\ &\geqslant \frac{1}{2M} \left(\frac{1}{M-1}\right)^{1/[3(M-1)]} (n_\Lambda^{\tilde{\eta}})^{1+1/[3(M-1)]} \end{aligned}$$

Together with the obvious inequality

$$p_\Lambda^{\tilde{\eta}} = \sum_{i=2}^{n} p_\Lambda^{\tilde{\eta}}(i) = \sum_{k=1}^{n_\Lambda^{\tilde{\eta}}} p_\Lambda^{\tilde{\eta}}(i_k) \leqslant (M-1)\, n_\Lambda^{\tilde{\eta}}$$

which estimates the number of points from the graph $\tilde{\eta}$ in cube Δ, it allows us to write that

$$
\begin{aligned}
\sqrt{(2p_\Delta^{\tilde{\eta}})!}\,\exp&\left[-\alpha\sum_{i=2}^{n}\frac{p_\Delta^{\tilde{\eta}}()}{p^{\tilde{\eta}}(i)}\operatorname{diam}\{\tilde{\eta}(i)\}\right]\\
&\leqslant \exp\left[p_\Delta^{(\tilde{\eta})}\ln(2p_\Delta^{\tilde{\eta}})-\alpha\sum_{i=2}^{n}\frac{p_\Delta^{\tilde{\eta}}(i)}{p^{\tilde{\eta}}(i)}\operatorname{diam}\{\tilde{\eta}(i)\}\right]\\
&\leqslant \exp\left[(M-1)\,n_\Delta^{\tilde{\eta}}\ln(2(M-1)\,n_\Delta^{\tilde{\eta}})\right.\\
&\qquad\left.-\frac{\alpha}{2M}\left(\frac{1}{M-1}\right)^{1/[3(M-1)]}(n_\Delta^{\tilde{\eta}})^{1+1/[3(M-1)]}\right]
\end{aligned}
\tag{A.2}
$$

It is easy to see that for any $M\geqslant 2$ the right-hand side of (A.2) becomes vanishingly small with $n_\Delta^{\tilde{\eta}}\to\infty$ uniformly with respect to Δ and $\tilde{\eta}$. ∎

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